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Weak and measure valued solutions of the Cauchy problem by using incompressible Euler equations

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Abstract

The paper concerned the existence problem for weak and measured valued solutions to the Cauchy problem by using the incompressible Euler equations . These are physically not admissible since their kinetic energy increases at least at the initial time. Every measure valued solutions is generated by a sequence of weak solutions and that therefore ,surprisingly, weak solutions are flexible as measured valued solutions. A common feature of these results is their relying on methods recently developed by De Lellis and L.Szekelyhidi. Jr. Where in particular global existence to any initial data was proven.

Keywords Young measure, Dissipative solutions, weak energy inequality ,Strong energy inequality, Local energy inequality, barycentre.

INTRODUCTION

In this paper we consider the Cauchy problem by using incompressible Euler equation of ideal fluid motion dimensions, A common feature of these results is their relying on methods recently developed by De Lellis and L.Szekelyhidi. Jr[1].

$$\partial_t v(x,t) + div(v(x,t)\otimes v(x,t)) + \nabla p(x,t) = 0$$

$$div\ v(x,t) = 0 \quad (1.1)$$

This non linear system of partial differential equations was derived by Leonhard Euler in 1757. It describes the motion of an incompressible fluid with velocity field and scalar pressure in the absence of external forces. Here, the time T can positive or infinity , denotes the matrix with entries and the divergence of us taken row-wise. The term " ideal" is synonymous with "inviscid",it refers to the absence of viscosity and thus the absence of effects of friction within the fluid.

A fundamental quantity in the study of the Euler equations is the kinetic energy ,

$$\frac{1}{2}\int_{\mathbb{R}^d} |x,t|^2\, dx$$

Suppose that v and p are a smooth solution of (1.1) and that v decays sufficiently fast at spatial infinity.

On the other hand, the existence of smooth solutions is unknown even for smooth initial data. What is known however ,is *local* existence; here, "local" refers to the time variable. In other words given smooth and sufficiently decaying initial data v_0 ,there exists a finite time $T > 0$ such that there exists a smooth solution on $\mathbb{R}^d \times [0,T]$,which is unique by the above argument. This was proved for the first time by L.Lichtenstein [2].

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A natural way to define such weak solutions is to integrate the given equation against a test function and then perform an integration by parts; more concretely, in the case of Euler, suppose first that v is a smooth solution of (1.1), and let $\phi \in C_c(\mathbb{R}^d \times [0,T))$ be a divergence-free vector field. Multiplication of (1.1) by ϕ , integration in space and time, and an integration by parts yield

$$\int_0^T \int_{\mathbb{R}^d} (v.\partial_t \phi + (v\otimes v):\nabla\phi)dxdt = 0 \quad (1.2)$$

where $A:B = \sum_{i,j} A_{ij}B_{ij}$ denotes the scalar product of two matrices. Similarly, for every function $\psi \in C_c^\infty(\mathbb{R}^d \times (0,T)]$ the incompressibility condition implies

$$\int_0^T \int_{\mathbb{R}^d} v.\nabla\psi\, dxdt = 0 \quad (1.3)$$

One says that $v \in L^2_{loc}(\mathbb{R}^d \times [0,T))$ is a weak solution to (1.1) if (1.2) and(1.3) are satisfied for every divergence-free $\phi \in C_c^\infty(\mathbb{R}^d \times (0,T)$ and every $\psi \in C_c^\infty(\mathbb{R}^d \times (0,T)$ respectively.

The discussion of measure-valued solutions will require some preparation. As mentioned before, a measure-valued solution is no longer a vector field, i.e. map $(x,t) \mapsto v(x,t)$, but a parameterized measure or Young measure, i.e. a map $(x,t) \mapsto v_{x,t}$, where for almost every x and t, $v_{x,t}$ is a probability measure on $\mathbb{R}^d$.

The intuition is that a measure-valued solution does not give the deterministic velocity of the fluid at a certain point in space-time, but only a probability distribution for the velocity. Such measures were introduced by L. C. Young[2,3] in order to study the relaxation of certain variation problems and have since then been employed as a useful tool in the calculus of variations and partial deferential equations .

In their setup, a generalized Young measure is a triple of measures, consisting of

• the oscillation measure (or classical Young measure) $v_{x,t}$ which is a probability measure on $\mathbb{R}^d$ for Lebesgue a.e. x and t;

• the concentration measure λ , which is a non-negative measure on $\mathbb{R}^d \times$[0; T];

• the concentration-angle measure $v_{x,t}^{\infty}$, which is a probability measure on the d − 1-dimensional unit sphere S^{d-1} for λ a.e. x and t.

The young measure satisfy,

$$\partial_t \langle v_{x,t}, \xi \rangle + div(\langle v_{x,t}, \xi \otimes \xi \rangle + \langle v_{x,t}, \theta \otimes \theta \rangle \lambda) + \nabla p = 0$$

$$div \langle v_{x,t}, \xi \rangle = 0 \qquad (1.4)$$

these equations is called a measure valued solutions of Euler equations.

Moreover, it is possible to define a meaningful notion of initial data and of kinetic energy of measured valued solutions. If the energy satisfies $E(t) \le \frac{1}{2}\int |v_0|^2 \, dx$ for a.e ,t. the measure valued solution is called *admissible* in analogy with admissible weak solutions.

2.Basic Notation

Let us fix some notation that we will use throughout this paper. We will denote by $M^{+}(X)$ and $M^{1}(X)$ the space of positive finite measures and probability measures on a measurable space X respectively, and for an open or closed subset $U \subseteq \mathbb{R}^m$, a positive Borel measure μ on U and an open or closed subset $V \subseteq \mathbb{R}^l$ we denote by $L_w^{\infty}(U, \mu, M^1(v))$ the space of μ -weakly*-measurable maps from U into $M^1(v)$ .We will denote by L_x^2 the space $L^2(\mathbb{R}^d)$,and by $L_t^{\infty} L_x^2$ and the space $L^{\infty}([0,T]; L^2(\mathbb{R}^d))$.

3.Weak solutions of Cauchy problem

In this section discussion about the definition and theorem of the weak solutions of incompressible Euler equations.

3.1 Definition

A Vector field $v \in L_{loc}^2(\mathbb{R}^d \times [0,T))$ is called a weak solution of the Cauchy problem for the Euler equations with initial data $v_0 .(v_0 \in L^2(\mathbb{R}^d)$ weakly divergence-free) if v is weakly divergence –free in the sense of,

$$\int_0^T \int$$

and if for every $\phi \in C_c(\mathbb{R}^d \times [0,T))$ with zero divergence

$$\int_0^T \int_{\mathbb{R}^d} (v.\partial_t \phi + (v \otimes v) : \nabla \phi) dx dt + \int_{\mathbb{R}^d} v_0(x) \phi(x,0) dx = 0$$

Local refers to "TIME "variable.

3.2 Definition

Distribution p such that v and p solve

$$\partial_t v(x,t) + div(v(x,t) \otimes v(x,t)) + \nabla p(x,t) = 0$$

The distributions,

$$\int_0^T \int_{\mathbb{R}^d} (v.\partial_t \psi + (v \otimes v) : \nabla \psi + p div \psi) dx dt = 0$$

Holds for every $\psi \in C_c^{\infty}(\mathbb{R}^d \times (0,T)]$. P is a distributional solution of $-\Delta p = div div(v \otimes v)$.

3.3 Definition

A Subsolution to the incompressible Euler equations With respect to the kinetics energy density is a triple,

$$(\bar{v}, \bar{u}, \bar{q}) : T^n \times (0,T) \to \mathbb{R}^n \times s_0^{n \times n} \times \mathbb{R}$$

With $\bar{v} \in L_{loc}^2$, $\bar{u} \in L_{loc}^{'}$, $\bar{q} \in D^{'}$,

such that,

$$\partial_t \bar{v} + div \; \bar{v} + \nabla q = 0$$

$$\text{div } v = 0$$

And $$\bar{v} \otimes \bar{v} - \bar{u} \le \frac{2}{n} \; \bar{e} \text{ a.e}$$

Here, $S_0^{n \times n}$ denotes the set of symmetric traceless n $\times$ n matrices. I is the identity matrix. Then v is weak solutions of Euler Equations.

3.4 Definition

If $v \in c([0,T] : L_w^2(\mathbb{R}^d))$ is a weak solution for Euler with initial data v_0 .It satisfies the weak Energy Inequality If for every $t > 0$,

$$\frac{1}{2}\int_{\mathbb{R}^d} |v(x,t)|^2 dx \le \frac{1}{2}\int_{\mathbb{R}^d} |v_0(x)|^2 \, dx$$

3.5 Definition

If in addition

$$\frac{1}{2}\int_{\mathbb{R}^d} \left| v(x,t)^2 dx \right| \le \frac{1}{2}\int_{\mathbb{R}^d} |v(x,s)|^2 \, dx$$

$\forall$ s,t With $t > s$ then v satisfies the strong Energy Inequality.

3.6 Theorem

Suppose $v_0 \in L^2(T^d)$ is weakly divergence free, for some $T > 0$. let $\bar{e} \in C([0,T]; L^{'}(T^d])$

$$\int \bar{e}(x,t) dx \le \int \frac{1}{2} |v_0(x)|^2 \, dx \quad \forall \; t \in [0,1)$$

Suppose there exists a sub solution $(\bar{v},\bar{u})$ with respect to $\bar{e}$ .And v_0 an open set such that $(\bar{v},\bar{u})$ the corresponding pressure $\bar{q}$ and $\bar{e}$ continuous on Ω, $\bar{U} \in C[0,T]; L^2_w(T^d)$ and $\bar{v}\otimes\bar{v}-\bar{u} < \frac{2}{d}$ $\bar{e}$ $I_d\Omega$ Then there exists infinitely many weak solutions to Euler with initial data v_0 and energy density $\bar{e}$.

3.7 Theorem

There exist infinitely many weak solution with vortex sheet initial data which satisfy the energy infinitely many which satisfy the strong energy inequality but not the energy equality.

Proof

For the notational simplicity, let us consider only the case d=2

$$S(\tau) = \begin{cases} 1 & if\ -\pi<\tau<0 \\ -1 & if\ \ 0<\tau<\pi \end{cases}$$

and extend periodically. Let $\lambda \in (0,1)$ and $\alpha = \alpha(x_2,t)$ be the entropy solution of Burger's equation,

$$\partial_t \alpha(x_2,t) + \frac{\lambda}{2}\partial_2(\alpha(x_2,t)^2) = 0$$

With initial data $\alpha(x_2,0) = s(x_2)$. α is explicit given by,

$$\alpha(x_2,t) = \begin{cases} -1 & if\ \ -\pi < x_2 < -\lambda t \\ \frac{x_2}{\lambda t} & if\ \ -\lambda t < x_2 < \lambda t \\ 1 & if\ \ \lambda t < x_2 < \pi \end{cases}$$

Up to time $T = \frac{\pi}{\lambda}$

Next, set $\beta = \beta(x_2,t) = \frac{1}{2}\alpha^2$ and $\gamma = \gamma(x_2,t) = -\frac{\lambda}{2}(1-\alpha^2)$

Then, a subsolution is given by,

$$\bar{v}(\alpha,0),\ \bar{u} = \begin{pmatrix} \beta & \gamma \\ \gamma & -\beta \end{pmatrix},\ \bar{q} = B$$

A simple calculation then shows that for $T = \frac{\pi}{\lambda}$ and

$$\bar{e} = \frac{1}{2} - e^{\frac{1-\lambda}{2}(1-\bar{\alpha}^2)}$$

where $\varepsilon \in [0,1]$. Is arbitrary $(\bar{v},\bar{u},\bar{q})$ satisfies the formulations.

By using above theorem,

The solutions obtained are energy conserving if $\varepsilon = 0$ and energy decreasing if $\varepsilon > 0$.

3.8 Definition

Let now v be a weak solution in $L^3_{loc}(\mathbb{R}^d \times (0,T))$. Then it satisfies the local energy inequality if,

$$\partial_t \frac{|v|^2}{2} + div((\frac{|v|^2}{2} + p)v) \le 0 .$$

In the sense of distributions.

3.9 Definition

A Dissipative solution of the Euler equations with initial data v_0 is a vector field $v \in L^\infty([0,\infty) : L^2(\mathbb{R}^d)$ with $v \in C([o,\infty) : L^2_w(\mathbb{R}^d)$ Such that

v(.,0) = v_0 ,div = 0

Weakly,and

$$\int_{\mathbb{R}^d} |v-u|^2 dx \le \exp(2\int_0^t \| d^-(u)\|_\infty\, ds) \int_{\mathbb{R}^d} |v.,0)-u(.,0)|^2$$

$$+2\int_0^t \int_{\mathbb{R}^d} \exp 2\int_s^t \| d^-(u)\|_\infty d\tau) E(u).(v-u)dxds$$

holds for every weakly divergence free vector field $u \in C([0,\infty); L^2(\mathbb{R}^d))$ for which $d(u) \in L^1_{loc}([0,\infty); L^\infty_x)$ and $E(u) \in L^1_{loc}([0,\infty); L^2_x$

3.10 Theorem

Let $\eta \in \mathbb{R}^{n+1}$ be a vector which is not parallel to e_{n+1} Then For any bounded open set $B \subset \mathbb{R}^n$

$$\lim_{n\to\infty} \int_B \sin^2(N\eta.(x,t))dx = \frac{1}{2}|B| \text{ uniformly } t \in \mathbb{R} .$$

Proof

Let us write $\eta = (\eta', \eta_{n+1}) \in \mathbb{R}^n \times \mathbb{R}$,so that $\eta' \in \mathbb{R}^n / \{0\}$ .by elementary trigonometric identities.

$$\sin^2(N\eta.(x,t) = \sin^2(N\eta'.x) + \sin^2(N\eta_{n+1}t)\cos(2N\eta'.x) + \frac{1}{2}\sin(2N\eta'.x)\sin(2N\eta_{n+1}t)$$

for the second term we have

$$|\int_B \sin^2(N\eta_{n+1}t)\cos(2N\eta'.x)dx|$$

$$\le \int_B \cos(2N\eta' x)dx| \to 0$$

as $N \to \infty$. And similarly third term vanishes in the limit uniformly in *t.*

3.11 Theorem

The functional $I_{\in,\Omega_0}$ are lower semi-continuous on X.

Proof

Assume, for a contradiction that exists $v_k, v \in X$ such that

$v_k \xrightarrow{d} v$ in X but,

$$\lim_{k\to\infty} \inf_{t\in[\in,T-\in]} \int_{\Omega_0} \left[\frac{1}{2}|v_k(x,t)|^2 - \bar{e}(x,t)\right]dx < \inf_{t\in[\in,T-\in]} \int_{\Omega_0} \left[\frac{1}{2}|v_k(x,t)|^2 - \bar{e}(x,t)\right]dx$$

Then there exists a sequence of times $t_k \in [\in, T-\in]$ such that

$$\lim_{k\to\infty} \int_{\Omega_0} \left[\frac{1}{2}|v_k(x,t_k)|^2 - \bar{e}(x,t_k)\right]dx <$$

$$\inf_{t\in[\in,T-\in]} \int_{\Omega_0} \left[\frac{1}{2}|v_k(x,t)|^2 - \bar{e}(x,t)\right]dx$$

We may assume without loss of generality that $t_k \to t_0$ .since the convergence in X is equivalent topology of $C([0,T];L_w^2)$ .We obtain the $v_k(.,t_k) \to v(.,t_0)$ in $L^2(\mathbb{R}^n)$ Weakly and hence $\lim_{k\to\infty} \int_{\Omega_0} \left[\frac{1}{2}|v_k(x,t_k)|^2 - \bar{e}(x,t_k)\right]dx \geq \int_{\Omega_0} \left[\frac{1}{2}|v(x,t_0)|^2 - \bar{e}(x,t_0)\right]dx$ a contradiction.

3.12 Theorem

For any $w \in s^n$ let $\lambda_{\max}^w$ denote the largest Eigen value of w. For $(v,u) \in \mathbb{R}^n \times S_0^n$

$$e(v,u) := \frac{n}{2}\lambda_{\max}(v \otimes v - u)$$

Then,

i) $e : \mathbb{R}^n \times S_0^n \to \mathbb{R}$ is convex.

ii) $\frac{1}{2}|v|^2 \leq e(v,u)$ with equality if and only if $u = v \otimes v - \frac{|v|^2}{n} I_n$.

iii) $|u|_\infty \leq 2\frac{n-1}{n} e(v,u)$ where $|u|_\infty$ denotes the operator norm of the matrix.

Proof

To proof (i)

Let $$e(v,u) = \frac{n}{2}\max_{\xi\in s^{n-1}} \langle \xi,(v\otimes v-u)\xi\rangle$$

$$e(v,u) = \frac{n}{2}\max_{\xi\in s^{n-1}} (\xi,\langle \xi,v\rangle v-u\xi)$$

Since for every $\xi \in s^{n-1}$ the map

$(v,u) \mapsto |\langle \xi,v\rangle|^2 - \langle \xi,u\xi\rangle$

is convex.

It follows that e is convex.

To prove (ii)

Since, $v \otimes v = v \circ v + \frac{|v|^2}{n} I_n$.

We have similarly to above that $e(v,u) = \frac{n}{2}\max_{\xi\in s^{n-1}} \langle \xi, v\circ v - u\xi\rangle|^2 + \frac{|v|^2}{2}$ Since $v \circ v - u$ is traceless, the sum of its Eigen value is zero. Therefore $\lambda_{\max}(v\circ v-u) \geq 0$. With equality if and only if $(v \circ v - u) = 0$ .Hence $\frac{1}{2}|v|^2 \leq e(v,u)$.

To prove (iii)

Let us take

$$e(v,u) = \frac{n}{2}\max_{\xi\in s^{n-1}}[|\langle \xi,v\rangle|^2 - \langle \xi,u\xi\rangle]$$

$$e(v,u) = \frac{n}{2}\lambda_{\max}(v\circ v-u) + \frac{|v|^2}{2}$$

We deduce, $e(v,u) = \frac{n}{2}\max_{\xi\in s^{n-1}}(-\langle \xi,u\xi\rangle]$

$$= \frac{n}{2}\lambda_{\min}(u)$$

Therefore $-\lambda_{\min}(u) \leq \frac{2}{n} e(v,u)$.

Since u is traceless, the sum of its Eigen value is zero. Then

$$|u|_\infty \leq (n-1)\lambda_{\min}(u)$$

$$\leq \frac{2(n-1)}{n} e(v,u)$$

4.measure valued solutions of Cauchy problem

In this section we discussed about the definition and theorem of measured valued solutions.

4.1 Definition

The Generalized young measure on $\mathbb{R}^{'}$ with parameters in Ω is now defined as (v,λ,v^∞) such that

$$v \in L_w^\infty(\Omega; M^{'}(\mathbb{R}^l)$$

$$\lambda \in M^+(\bar{\Omega})$$

And $v^\infty \in L_w^\infty(\bar{\Omega},\lambda; M^{'}(s^{l-1}))$.

where, $M^{'}(x)$ Probability measure.

$M^+(x)$ is positive finite measure.

4.2Definition

The oscillation measure $v_{x,t}$ which is probability measure on

$\mathbb{R}^d$ for lebesgue a.e x and t $\mathbb{R}^d$

for lebesgue a.e x and t .

$$v \in L_w^\infty(\Omega; M^{'}(\mathbb{R}^l)$$

4.3 Definition

The concentration measure λ , which is a non-negative measure on $\mathbb{R}^d \times [0,T]$,

$$\lambda \in M^+(\bar{\Omega})$$

4.4 Definition

The Concentration Angle measure $v_{x,t}^\infty$ which is a probability measure on the d-1 dimensional unit sphere S^{d-1} for λ a.e x and t,

$$v^\infty \in L_w^\infty(\bar{\Omega},\lambda; M^{'}(s^{l-1}))$$

4.5 Theorem

Let $v_n(x,t)$ be a sequence of a functions $\mathbb{R}^d \times [0,T] \to \mathbb{R}^l$ which is bounded in

$L^{\infty}[0,T];L^2(\mathbb{R}^d)$ and generates a young measure (v,λ,v^{∞}) in $L^2(\mathbb{R}^d)\times[0,T]$. Then

$$ess\sup_t(\int_{\mathbb{R}^d}\langle v_{x,t},|.|^2\rangle dx<\infty$$

And the concentration measure λ admits a disintegration measure

$\lambda(dxdt)=\lambda_t(dx)\otimes dt$,where $t\to\lambda_t$ is bounded measurable map form $\mathbb{R}^d\times[0,T]$ into $M^+(\mathbb{R}^d)$.

Proof

A measure on [0,T] by $u(A):=\lambda(\mathbb{R}^d\times A)$ for a Boral subset $A\subseteq[0,T]$.

By standard measure theory there exists a measureable map $t\to\bar{\lambda}_t$ with

$\bar{\lambda}_t\in M^+(\mathbb{R}^d)$

Such that $\lambda(dxdt)=\bar{\lambda}_t(dx)*\mu(dt)$

$$\|v_n\|_{L_x^2}^2(t)dt\overset{*}{\to}\left(\int_{\mathbb{R}^d}\langle v_{x,t},|.|^2\rangle dx\right)dt+\mu dt$$

which means that for every $\phi\in C_c[0,T],\phi\geq 0$,

$$\int_0^T\phi(t)\|v_n\|_{L_x^2}^2(t)dt\to\int_0^T\phi(t)\left(\int_{\mathbb{R}^d}\langle v_{x,t},|.|^2\rangle dx\right)dt+\int_0^T\phi(t)\mu dt$$

Hence,

$$\left|\int\phi\left(\int_{\mathbb{R}^d}\langle v_{x,t},|.|^2\rangle dx\right)dt\right|\leq\sup_n\left|\int\phi\|v_n\|_{L_x^2}^2dt\right|$$

$$\leq\sup_n\int\|v_n\|_{L_x^2}^2\|\varphi\|_{L^{'}([0,T])}dt$$

From the which it follows that

$ess\sup_t(\int_{\mathbb{R}^d}\langle v_{x,t},|.|^2\rangle dx<\infty$ Similarly,

$$\left|\int\phi\mu(dt)\right|\leq\sup_n\int\|v_n\|_{L_x^2}^2\|\varphi\|_{L^{'}([0,T])}dt$$

Hence by the $\mu=h(t)\,dt$, again $h\in L^{\infty}([0,T])$.

So, by setting $\lambda_t=h(t)\bar{\lambda}_t$, obtain the disintegration of the form

$\lambda(dxdt)=\lambda_t(dx)\otimes dt$, Hence

$$ess\sup_t(\int_{\mathbb{R}^d}\langle v_{x,t},|.|^2\rangle dx<\infty$$

4.6 Definition

A measure valued solutions to the Euler equations is now a generalized young measure on $\mathbb{R}^d$ with parameters in $\mathbb{R}^d\times[0,T]$ which satisfies the Euler equations in an average sense .

The young measure satisfy,

$$\partial_t\langle v_{x,t},\xi\rangle+div(\langle v_{x,t},\xi\otimes\xi\rangle+\langle v_{x,t},\theta\otimes\theta\rangle\lambda)+\nabla p=0$$

$$div\langle v_{x,t},\xi\rangle=0$$

in the sense of distributions. Here the quantity

$$\bar{v}(x,t):=\langle v_{x,t},\xi\rangle$$

is called the barycentre of $v_{x,t}$.

4.7 Theorem

Let (v,λ,v^{∞}) be an admissible measure valued solution of the Euler equations and $\bar{v}$ its barycentre. Then

$$\bar{v}(.,t)\to\bar{v}(.,0)=v_0$$

Strongly in $L^2(\mathbb{R}^d)$ as $t\to 0$.

Proof

Given (v,λ,v^{∞}) be an admissible measure valued solution of the Euler equations.

Let $\bar{v}\in CL_w^2$, $\bar{v}$ its barycentre .and therefore

$$\liminf_{t\to 0}\|\bar{v}(t)\|_{L^2}\geq\|\bar{v}(0)\|_{L^2}.$$

On the other hand,

$$\int|\bar{v}(t)|^2dx=\int|\langle v_{x.t},\xi\rangle|^2dx$$

$$\int|\bar{v}(t)|^2dx\leq\int|\langle v_{x.t},\xi\rangle|^2dx+\lambda_t(\mathbb{R}^d)$$

$$=2E(t)$$

$$\leq\int|\bar{v}(0)|^2dx$$

where used the weak energy inequality of the admissible measure -valued solution. Combining the both inequalities yields

$$\|\bar{v}(t)\|_{L^2}\to\|\bar{v}(0)\|_{L^2}\ as\ \ t\to 0$$

and weak convergence together with convergence of the norms implies strong converge. Hence

$$\bar{v}(.,t)\to\bar{v}(.,0)=v_0.$$

strongly in $L^2(\mathbb{R}^d)$ as $t\to 0$.

4.8 Theorem

Suppose $w_n=(v_n,u_n)$ is a sequence bounded in $L^{\infty}([0,T]);L^2\times L^{'}(\mathbb{R}^d))$ and $f\in F_{2,1}$ Then there exists a subsequence (w_n) and a young measure (v,λ,v^{∞}). With

$v\in L_w^{\infty}(\mathbb{R}^d\times[0,T];M^{'}(\mathbb{R}^d\times S_0^d))$

$\lambda\in M^+(\mathbb{R}^d\times[0,T])$, $v^{\infty}\in L_w^{\infty}(\mathbb{R}^d,\lambda;M^{'}(s))$ such that for all $f\in F_{2,1}$

$$f(x,t;w_n(x,t))dxdt \to \langle v_{x,t}, f(x,t)\rangle dxdt + \langle v^{\infty}_{x,t}, f^{\infty}(x,t,.)\rangle \lambda$$

in the sense of measures.

4.9 Definition

The Lift Young measure from $\mathbb{R}^d$ to $\mathbb{R}^d \times S^d_0$ Define a map by

$$Q(\xi) = (\xi, \xi \circ \xi).$$

CONCLUSION

In this work we discussed about the weak solutions of the Cauchy problem using strong, weak and local energy inequalities and the measured valued solutions of the incompressible Euler equations by using the many type of measures.

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